Modified Gravity

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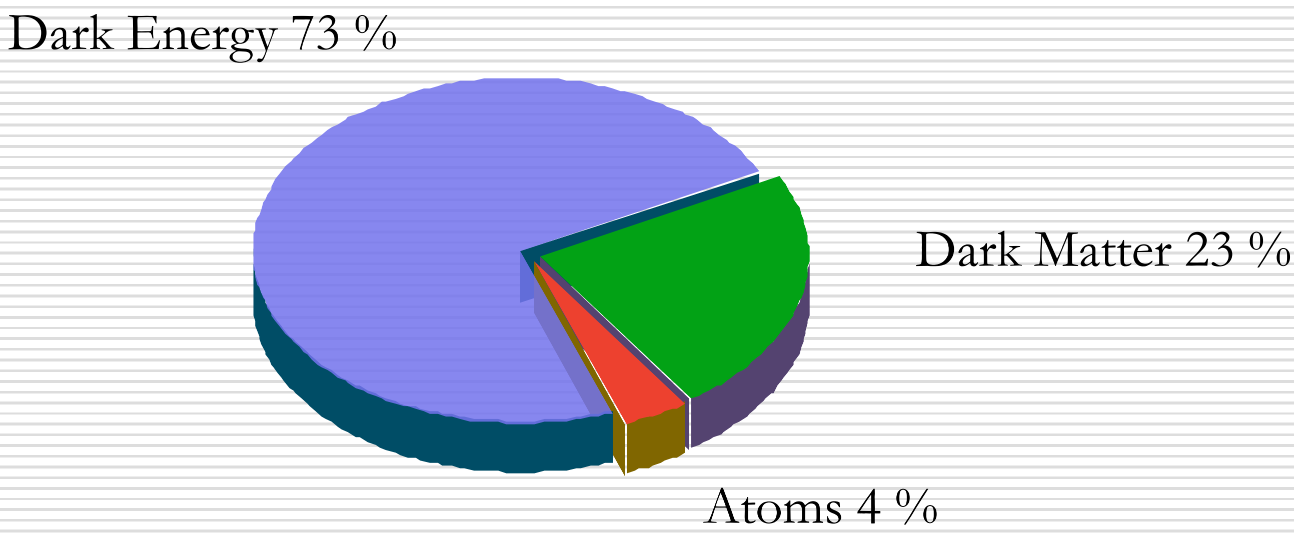
Outline

- Introduction
- Scalar-tensor theories
- Modified gravity –f(R) theories
- The Chameleon Mechanism
- Tests and Constraints
- Predictions
- Structure Formation with f(R) theories
- Fundamental physics?

Energy budget of universe



Expansion of universe accelerates rather than slowing down!

Fundamental questions raised:

- What is dark energy?
- Why have dark matter and dark energy roughly the same energy density?

Pressure and equation of state:

Strong energy condition: ½> 0, that means 3p < ¡ ½

So, for the equation of state we have:

$$w = \frac{p}{½} < ¡\ \frac{1}{3}$$

Observations (z=0): $\omega \approx -0.98$

$$\rho_{DE} = (10^{-3}\ \mathrm{eV})^4$$

What is the origin of this strange energy scale?

Problem: $$\rho_{DE} \approx 10^{-120} (10^{19}\,\mathrm{GeV})^4$$

<u>**This is the so-called cosmological constant problem.**</u>

Suitable candidates?

- *Cosmological constant.* **Plus:** works perfect! **Minus:** badly motivated from particle physics
- *Scalar Fields.* Can explain observations, motivated from particle physics; but many candidates strange properties, in general there is an initial condition problem in the very early universe. Coupling to matter leads to fifth forces.

Uncoupled scalar fields

Klein-Gordon-equation (scalar field-dynamics):

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

Potential-Term

Potential V = ??

Examples (Quintessence):

Exponential:

$$V = V_0 e^{\lambda \phi}$$

(Wetterich 1988; Wetterich 1995; Ferreira & Joyce (1997); Copeland, Liddle & Wands (1998))

Power-law:

$$V = \frac{M^{4+n}}{\phi^n}\,; \qquad n > 0$$

(Ratra & Peebles (1988); Caldwell, Dave & Steinhardt (1998); Binetruy (1999))

Hybrid:

$$V = \frac{M^{4+n}}{\phi^n} e^{\lambda \phi^2}$$

(Brax & Martin (2000,2001))

3.Do Scalars Couple to Matter? --- problems

Effective field theories with gravity and scalars

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R-\frac{1}{2}(\partial\phi)^2-V(\phi)+\mathcal{L}_m(\psi_m,A^2(\phi)g_{\mu\nu})\right)$$

deviation from Newton's law

$$\alpha=\frac{\partial\ln A}{\partial\phi}$$

Equations of motion („Newton“):

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = -\alpha\frac{\partial\phi}{\partial x_\mu}$$

$\Gamma^\mu_{\nu\lambda}$: Christoffel-symbols

$$\alpha = \frac{\partial \ln A(\phi)}{\partial\phi}$$

$\Rightarrow$

Scalar field transmits new force between particles!

Experimental consequences?

Long lived scalar fields which couple with ordinary matter lead to the presence of a new Yukawa interaction:

$$F_{12} = \frac{G_N m_1 m_2}{r^2}(1 + \alpha_1 \alpha_2 e^{-mr})$$

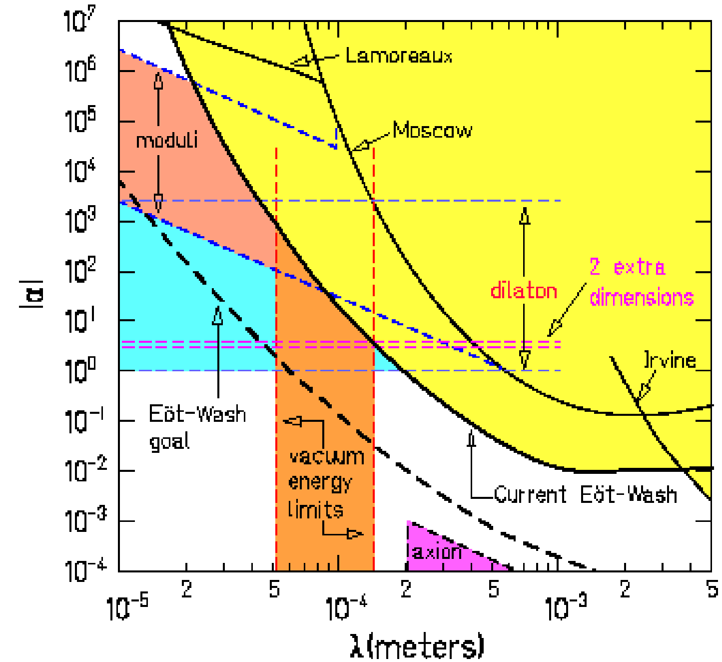
This new force would have gravitational effects on the motion of planets, the laboratory tests of gravity etc..

Gravity Tests

- Fifth force experiments

$$F = (1 + 2\theta) F_N$$

$$|\theta| \leq 5.10^{-5}$$



- Equivalence principle

$$\eta = \frac{|a_1 - a_2|}{2|a_1 + a_2|} \leq 10^{-12}$$

f(R) gravity

- The simplest modification of General Relativity is f(R) gravity:

$$\frac{1}{16\pi G_N}\int d^4x\sqrt{-g}R \rightarrow \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R)$$

- The function f(R) must be close to R, so f(R)= R+ h(R), h<< R in the solar system.
- f(R) gravity addresses the dark energy issue for certain choices of h(R).

f(R) vs Scalar-Tensor Theories

f(R) totally equivalent to an effective field theory with gravity and scalars

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\mathrm{Pl}}}g_{\mu\nu})\right)$$

The potential V is directly related to f(R).

$$V(\phi) = m_{\mathrm{Pl}}^2\frac{Rf' - f}{2f'^2}, \quad f' = e^{-2\phi/\sqrt{6}m_{\mathrm{Pl}}}$$

Same problems as dark energy: coincidence problem, cosmological constant value etc…

$$\alpha_\phi = \frac{1}{\sqrt{6}}$$

- The properties of **scalar-tensor** theories are **very well constrained** by lab and solar system tests of gravity.

- In many theories the **mass of the scalar field** is **approximately constant**:

$$\mathrm{mass}^2 = m_\phi^2 = V_{,\phi\phi} \approx \mathrm{const}$$

- Lab tests of l **re Law** give :

$$\beta \sim \mathcal{O}(1),\ \ m_\phi \gtrsim \frac{1}{50\,\mu\,\mathrm{m}}$$

  - **Cosmologically**, the scalar field would be so **heavy** that it would simply **be fixed to the minimum of its potential**.
  - It would **look just like a cosmological constant**.

Chameleons

Chameleon field: field with a matter dependent mass

A way to reconcile gravity tests and cosmology:

- Nearly massless field on cosmological scales
- Massive field in the atmosphere
- Allow large gravitational coupling constant of order one or more
- Possible non-trivial effects in the solar system (satellite experiments)

Chameleons

Chameleon field: field with a matter dependent mass

A way to reconcile gravity tests and cosmology:

Nearly massless field on cosmological scales

Massive field in the laboratory

Chameleon Theories

- For a chameleon theory one needs:

$$V_{,\phi}(\phi) < 0 \quad V_{,\phi\phi}(\phi) > 0, \quad V_{,\phi\phi\phi} < 0$$

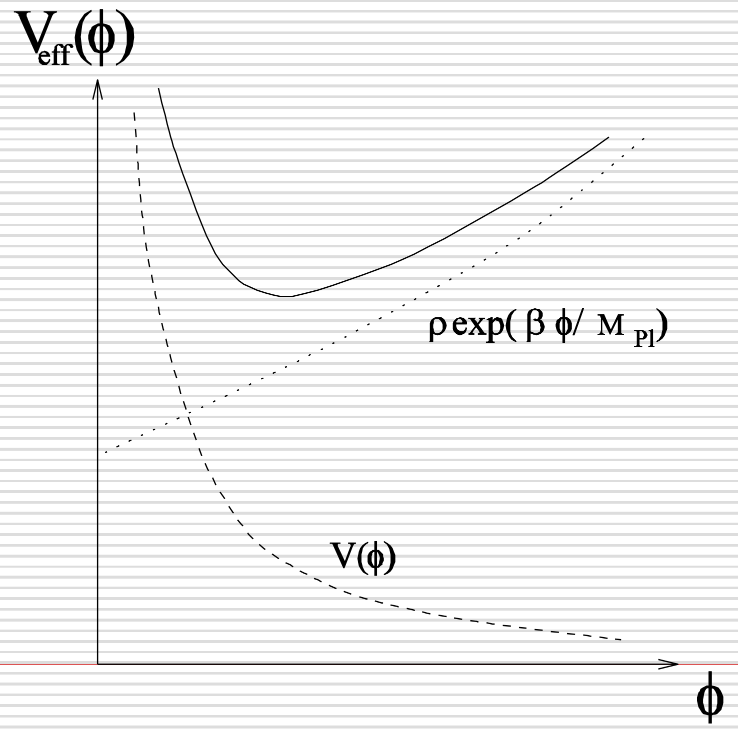
- In a background of non-relativistic matter, the field equation for the chameleon is:

$$\Box\phi = V^{\mathrm{eff}}_{,\phi}(\phi) = V_{,\phi} + \frac{\beta\rho}{M_{\mathrm{pl}}}$$

The Chameleon Mechanism

When coupled to matter, scalar fields have a matter dependent effective potential:

$$V_{eff}(\phi) = V(\phi) + \rho_m A(\phi)$$



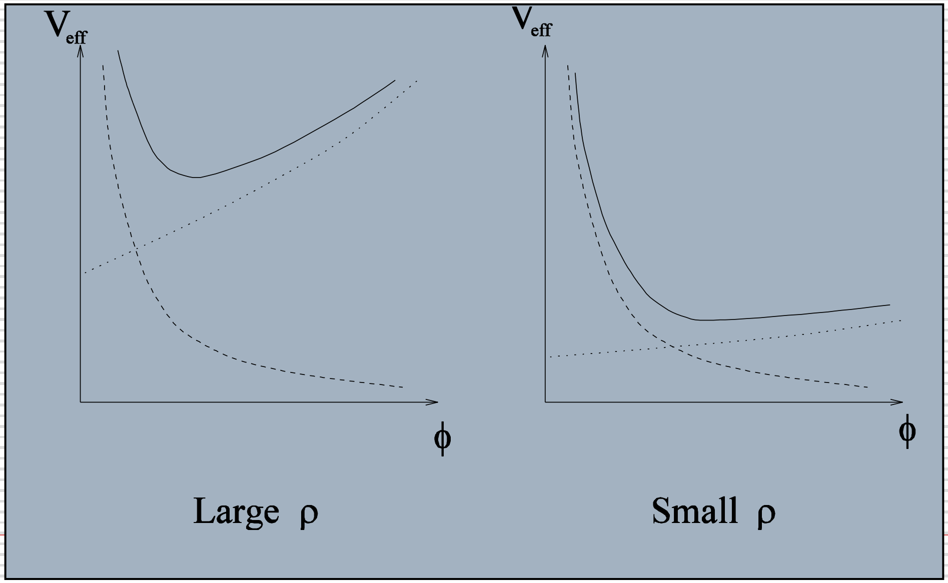
Typical example:

Ratra-Peebles potential

$$V(\phi) = \frac{M^{4+n}}{\phi^n}$$

Constant coupling to matter

$$A(\phi) = \exp \beta \frac{\phi}{m_p}$$



What is dense enough?

- The environment dependent mass is enough to hide the fifth force in dense media such as the atmosphere, hence no effect on Galileo's Pisa tower experiment!

$$\rho \approx 10^{-4} \mathrm{g/cm^3}$$

- It is not enough to explain why we see no deviations from Newtonian gravity in the lunar ranging experiment

$$\rho \approx 10^{-22} \mathrm{g/cm^3}$$

- It is not enough to explain no deviation in laboratory tests of gravity carried in "vacuum"

$$\rho \approx 10^{-14} \mathrm{g/cm^3}$$

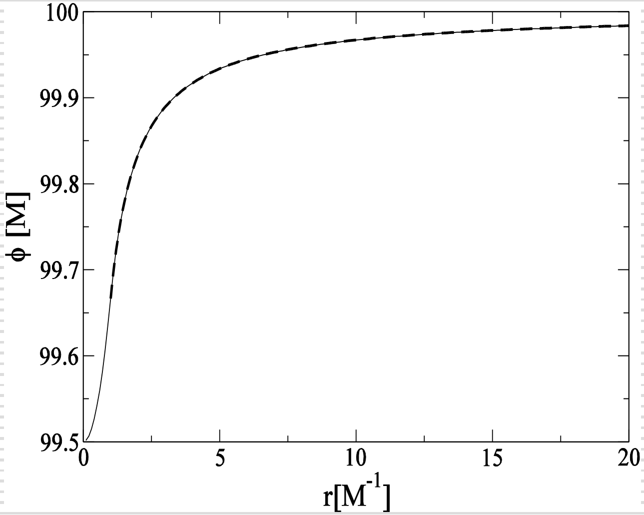
The Thin Shell Effect I

- The force mediated by the chameleon is:

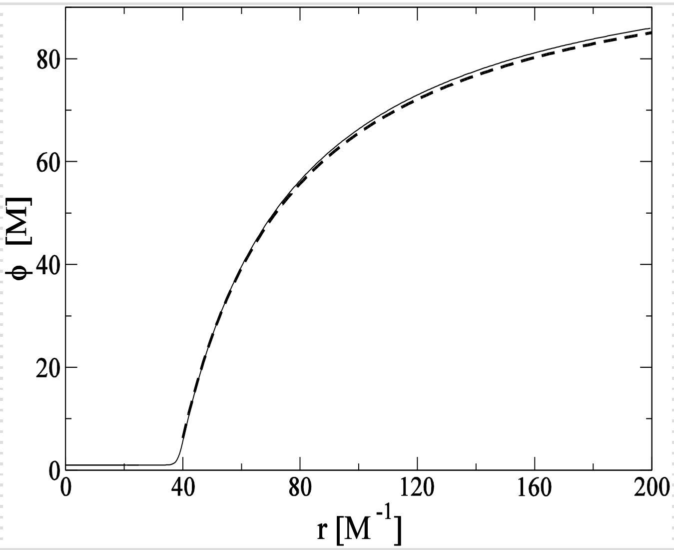
$$F_\phi = -\beta \frac{m}{m_{\mathrm{Pl}}} \nabla \phi, \quad \beta = \frac{m_{\mathrm{Pl}}}{M}$$

- The force due to a compact body of radius R is generated by the gradient of the chameleon field outside the body.
- The field outside a compact body of radius R interpolates between the minimum inside and outside the body
- Inside the solution is nearly constant up to the boundary of the object and jumps over a thin shell ΔR
- Outside the field is given by:

$$\phi \approx \phi_\infty - \frac{\beta}{m_p} \frac{3 \Delta R}{R} \frac{M_c}{r}$$



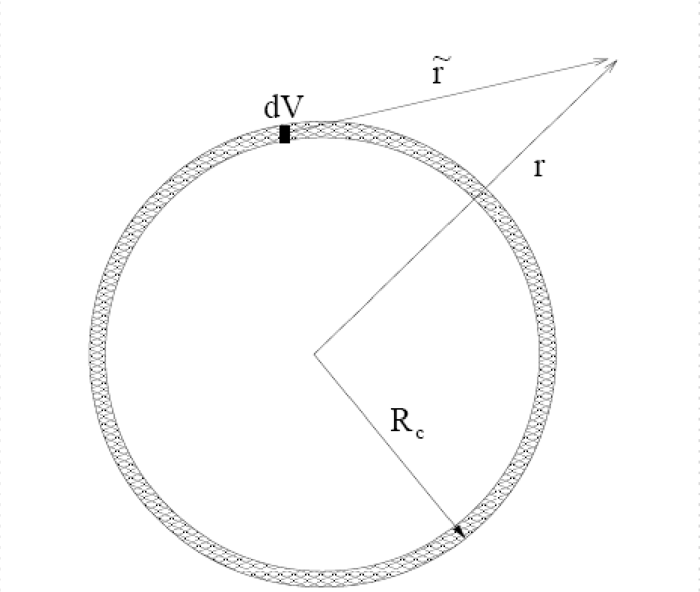
No shell



Thin shell

The thin-shell property

- The chameleon force produced by a massive body is due only to a thin shell near the surface



$$\theta = 2\beta^2$$

Khoury & Weltman (2004)

The Thin Shell Effect II

- The force on a test particle outside a spherical body is shielded:

$$\alpha_\phi = 3\beta \frac{\Delta R}{R}$$

- When the shell is thin, the deviation from Newtonian gravity is small.

- The size of the thin-shell is:

$$\frac{\Delta R}{R} = \frac{\phi_\infty - \phi_c}{6\beta m_p \Phi_N}$$

- Small for large bodies (sun etc..) when Newton's potential at the surface of the body is large enough.

f(R) gravity and chameleon theories

- Many authors have tried to employ a **chameleon mechanism to construct f(R) gravity theories.**

f(R) chameleon theories

- Some examples of chameleonic f(R) theories from the literature:
  - Hu & Sawicki: $$f(R) = R - \frac{R_1}{3}\frac{c_1\left(\frac{R}{R_1}\right)^n}{1+c_2\left(\frac{R}{R_1}\right)^n}$$
  - Starobinsky: $$f(R) = R + \lambda R_1\left(\left(1+\frac{R^2}{R_1^2}\right)^{-n} - 1\right)$$
  - Appleby & Battye: $$f(R) = \frac{R}{2} + \frac{R_1}{4b}\ln\left(\cosh\frac{2bR}{R_1} - \tanh(b)\sinh\frac{2bR}{R_1}\right),\ b \gg 1$$
  - In all cases ***$R_1 \sim H^2$*** today.

Laboratory tests

- In a typical experiment, one measures the force between two test objects and compare to Newton's law. The test objects are taken to be small and spherical. They are placed in a vacuum chamber of size L.
- In a vacuum chamber, the chameleon "resonates" and the field value adjusts itself according to:

$$m_{\text{vac}} L \sim 1$$

- The vacuum is not dense enough to lead to a large chameleon mass, hence the need for a thin shell.

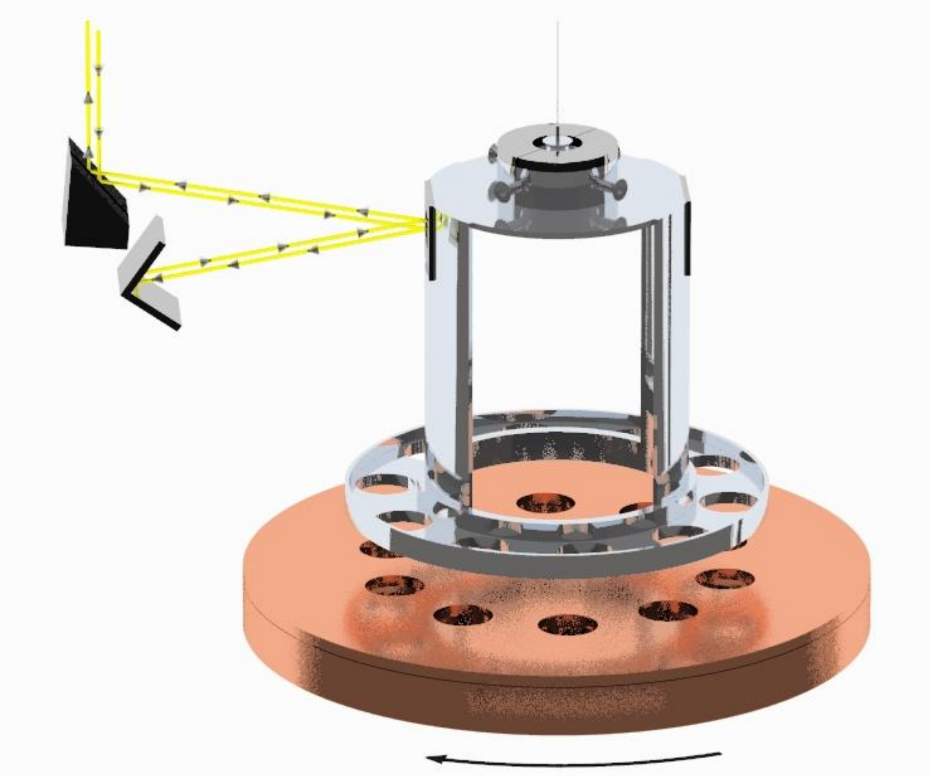
$$\phi_{\text{vac}} \leq 10^{-28} m_{\text{Pl}}$$

- Typically for masses of order 40 g and radius 1 cm, the thin shell requires for the Ratra-Peebles case:

$$\Lambda \leq 10^{3n/(n+4)} 10^{-12}\ \text{GeV}$$

Inverse Square Law Tests

- The most accurate current test of the Inverse Square Law (ISL) is the **Eot-Wash**



The attractor and detector plates have regular patterns of holes in them.

The separation between the plates is much smaller than the hole size.

If the ISL is violated the lower plate induces a torque on the upper plate.

Chameleons & ISL tests

- We've shown before that **the chameleonic force per unit area** between **two thin-shelled nearby plates** with separation ***d*** behaves as:

$$F_\phi/A \propto d^{-p}$$

- But need to **calculate the Torque for the Eot-Wash** experiment: complicated, although tractable, process.

- To make **accurate quantitative predictions** requires as **choice of *f(R)*.**

Choice of f(R)

- We note that when R >> R_1 both the **Hu-Sawicki** and **Starobinsky** forms of f(R) **have the form**:

$$f(R) \approx R + \frac{p}{p+1}\bar{R}\left(\frac{R}{\bar{R}}\right)^{p+1} \qquad \bar{R} \sim \mathcal{O}(R_1)$$

Where

- In Hu-Sawicki case: p = -1 –n,
- In Starobinsky case: p = -1-2n.
- This gives a **chameleon theory for all p < 1**

ISL bounds on f(R) theories

- For an f(R) theory to evade the Eot-Wash constraints, the **plates must have thin-shells** & the **torque** between two thin-shelled plates **must be small enough**.

- We also consider the constraints placed by the **naive Compton condition** i.e.

$$m_p D_p \gg 1$$

- Where m_p is the chameleon mass at the minimum of the effective potential inside the plates and D_p is the plate thickness.

ISL bounds on f(R) theories

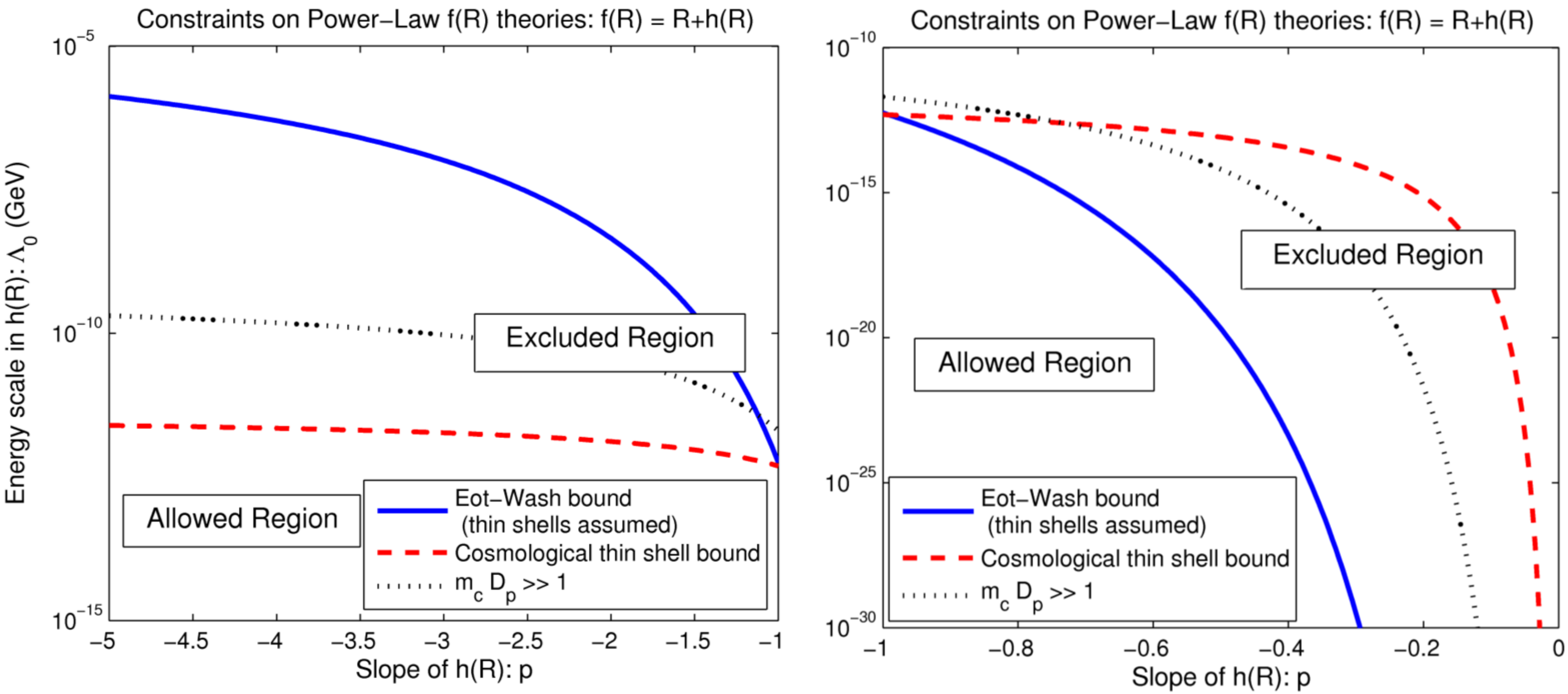
- We define

$$\Lambda_0^4 = M_{\rm pl}^2 \bar{R}$$

- If $\bar{R} \approx H_0^2, \quad \Lambda_0 \approx 2.4 \times 10^{-12}\,\mathrm{GeV}$

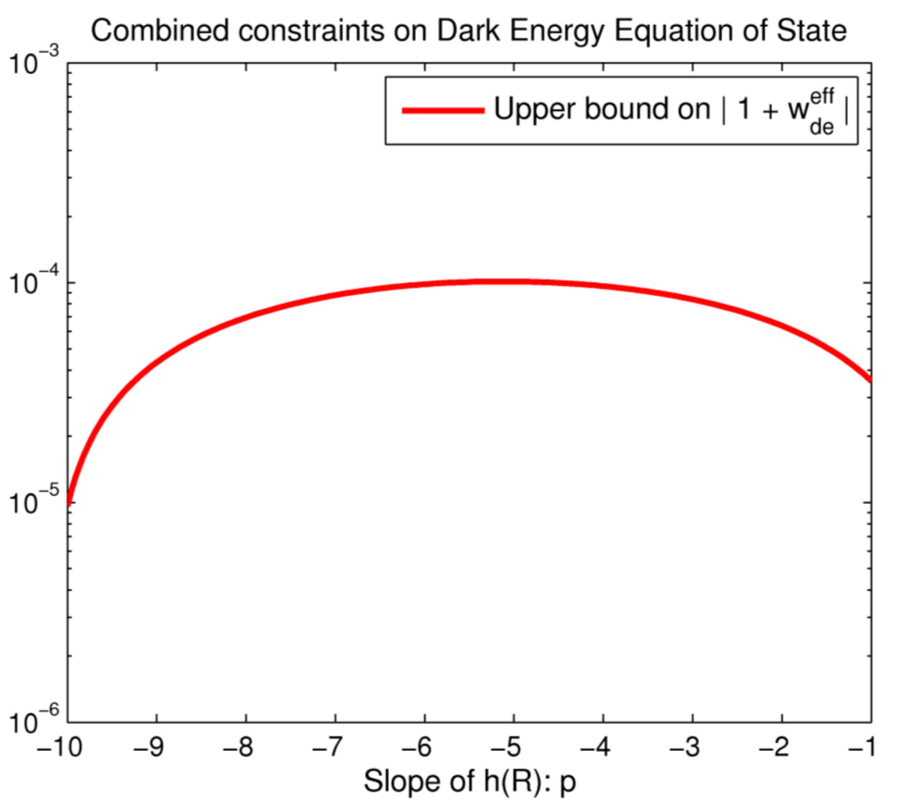
- We find that in **all cases this is ruled out**

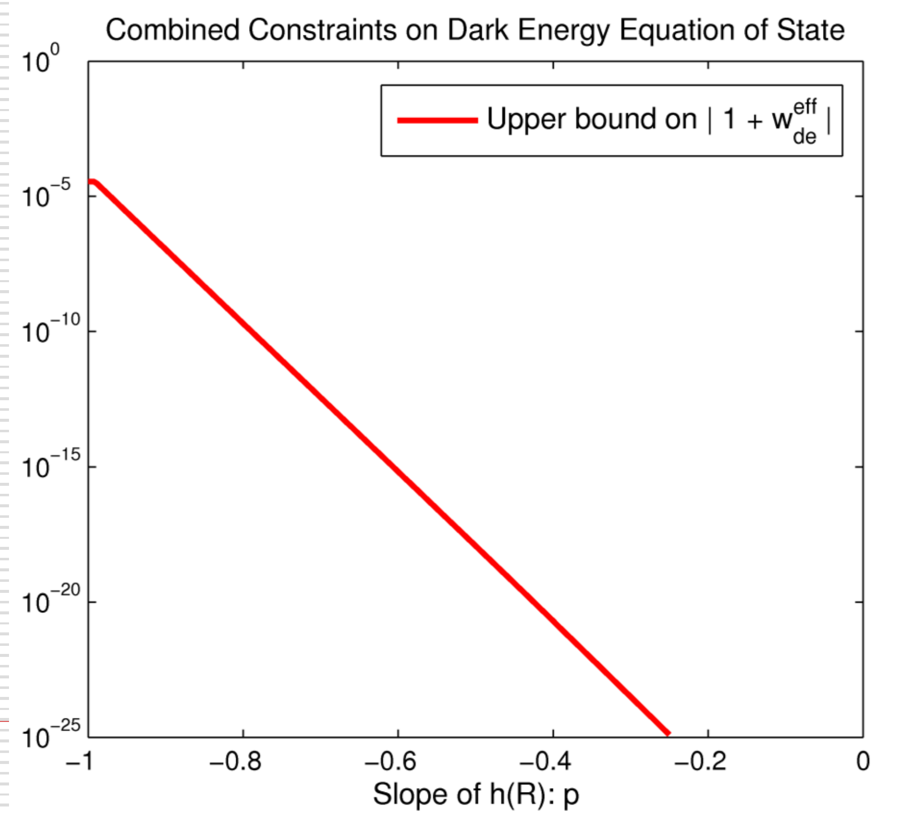
ISL bound on f(R) theories



Equation of State Bounds

- This give **tight bounds on any deviations** of the effective dark energy **Equation of State from -1**:

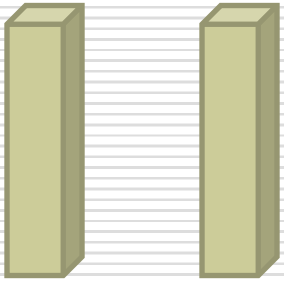




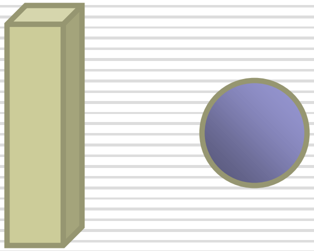
The Casimir Effect

Casimir Force Experiments

- Measure force between
  - Two parallel plates



  - A plate and a sphere



The Casimir Force

- The inter-plate force is in fact the contribution from a chameleon to the Casimir effect. The acceleration due to a chameleon is:

$$a_\phi = -\alpha\kappa_4\nabla\phi$$

- The attractive force per unit surface area is then:

$$\frac{F_\phi}{A} = -\int_{d/2}^{D+d/2} \alpha\kappa_4\rho_c \frac{d\delta\phi}{dx} = V'(\phi_c)\delta\phi_s$$

where

$$\delta\phi_s = \frac{V(\phi_b) - V(\phi_0) - V'(\phi_b)(\phi_b - \phi_0)}{V'(\phi_c)}$$

is the change of the boundary value of the scalar field due to the presence of the second plate.

The Casimir Force

- We focus on the plate-plate interaction in the range:

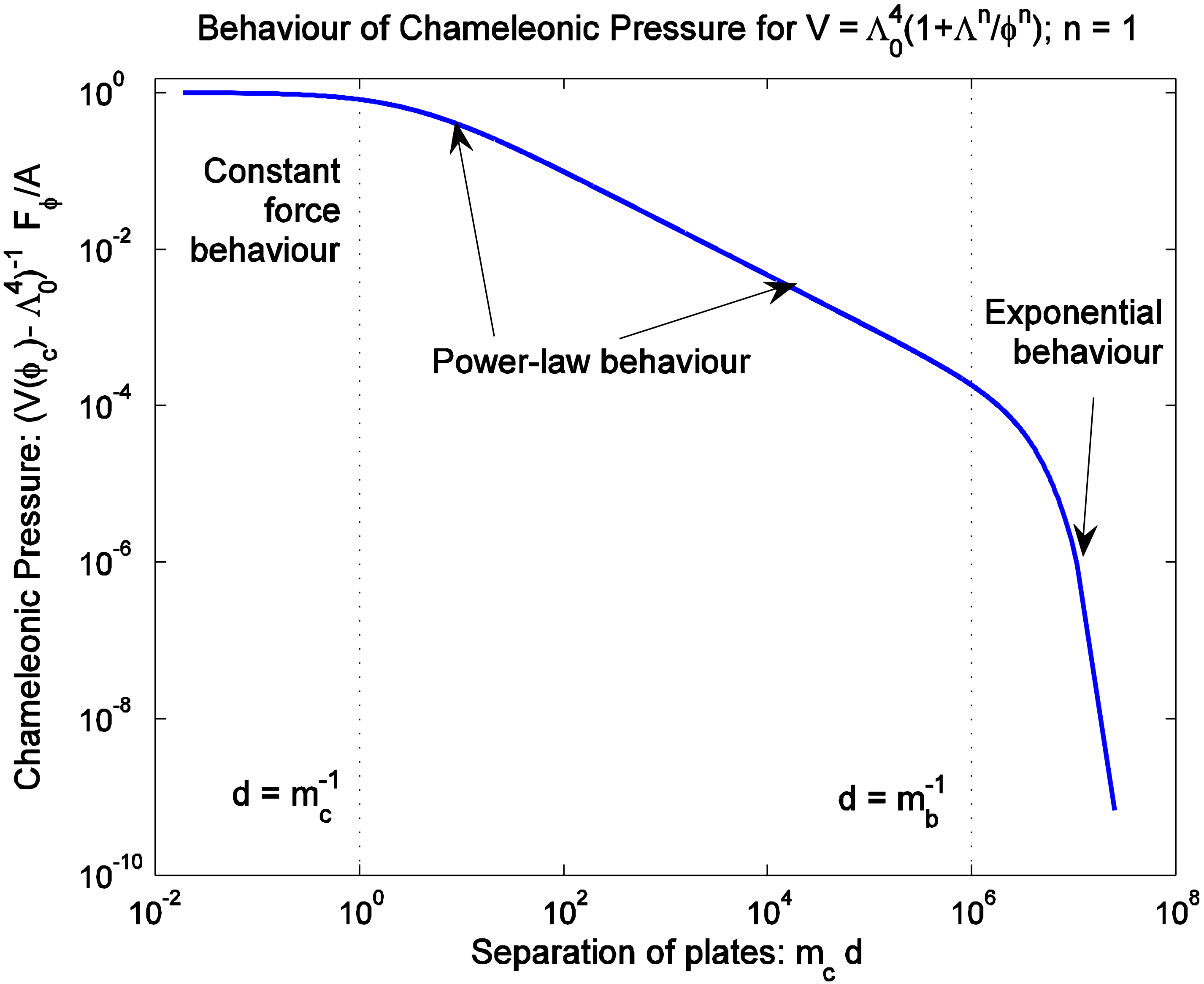
Mass in the plates → $$m_c^{-1} \leq d \leq m_b^{-1}$$ ← Mass in the cavity

- The force is algebraic:

$$\frac{F_\phi}{A} \sim \Lambda^4 (\Lambda d)^{-\frac{2n}{n+2}}$$

- The dark energy scale sets a typical scale:

$$\Lambda^{-1} \sim 82\mu m$$

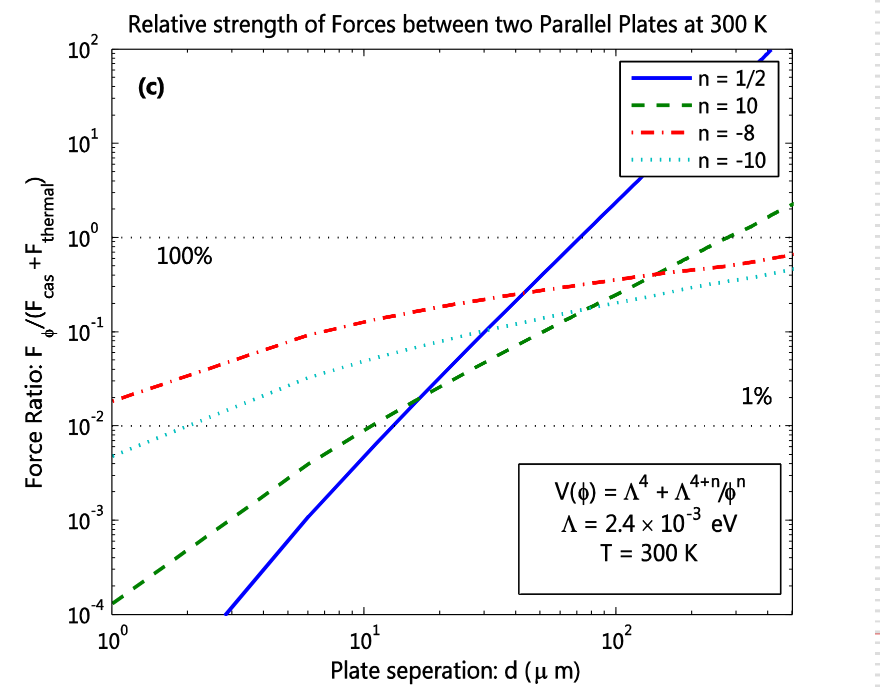

Behaviour of Chameleonic Pressure for $V = \Lambda_0^4(1+\Lambda^n/\phi^n)$; n = 1
Chameleonic Pressure: $(V(\phi_c)-\Lambda_0^4)^{-1}\ F_\phi/A$
Separation of plates: $m_c\, d$
Constant force behaviour
Power-law behaviour
Exponential behaviour
$d = m_c^{-1}$
$d = m_b^{-1}$
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
10^{8}

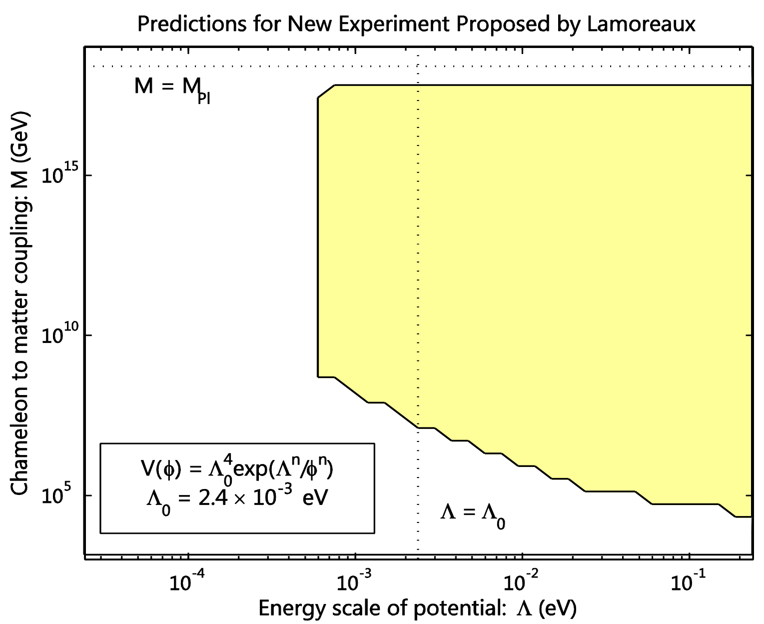
Detectability

- The Casimir forces is also an algebraic law implying:

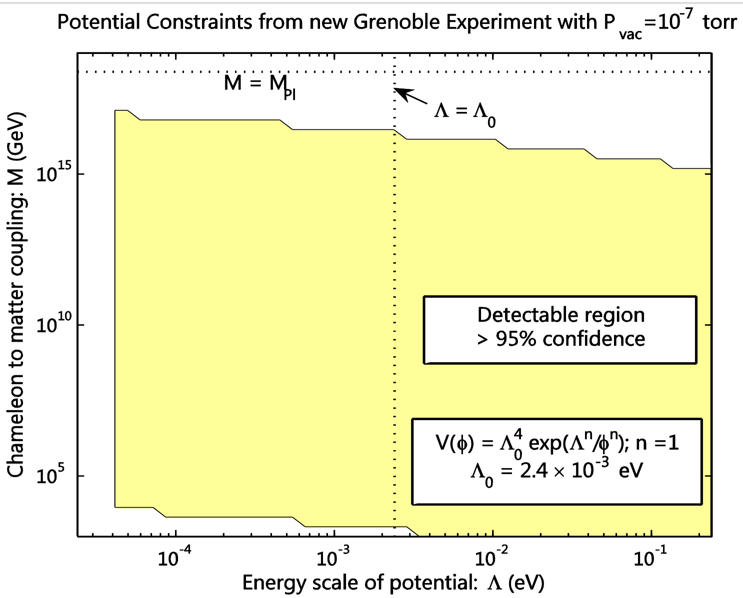
$$\frac{F_\phi}{F_{\text{cas}}} \sim \frac{240}{\pi^2}(\Lambda d)^{\frac{2(n+4)}{n+2}}$$

- This can be a few percent when d=10μm and would be 100% for d=30 μm

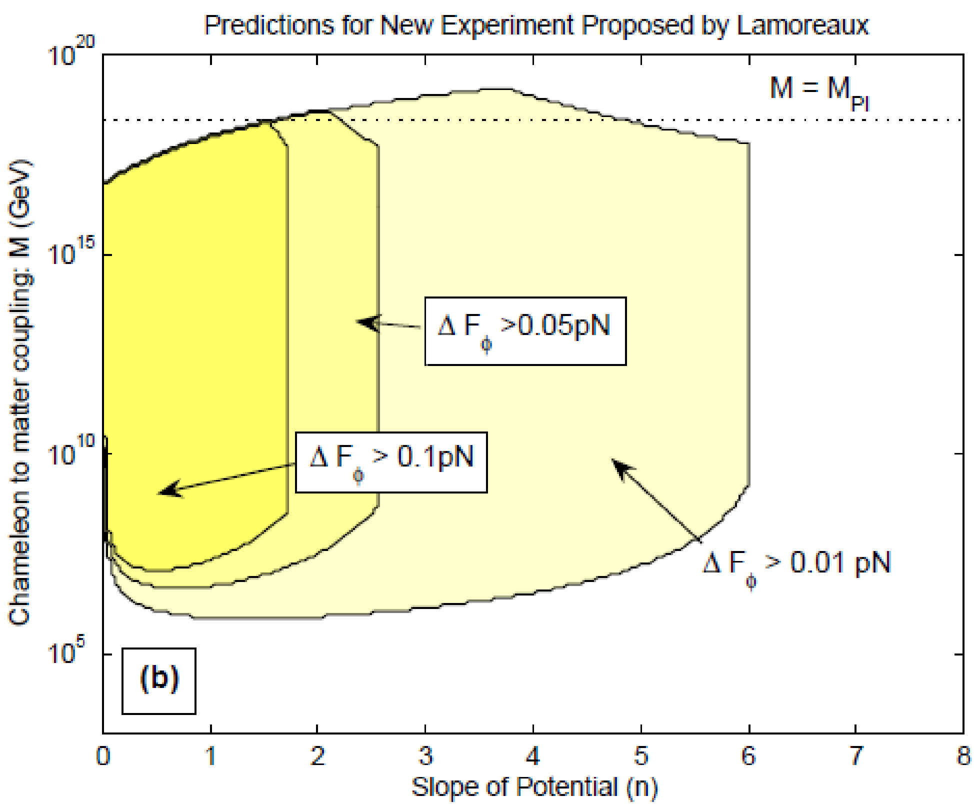




Predictions for New Experiment Proposed by Lamoreaux
M = M_Pl
Chameleon to matter coupling: M (GeV)
10^15
10^10
10^5
V(ϕ) = Λ_0^4 exp(Λ^n/ϕ^n)
Λ_0 = 2.4 × 10^-3 eV
Λ = Λ_0
10^-4
10^-3
10^-2
10^-1
Energy scale of potential: Λ (eV)



Potential Constraints from new Grenoble Experiment with P_vac = 10^-7 torr
M = M_Pl
Λ = Λ_0
Chameleon to matter coupling: M (GeV)
10^15
10^10
10^5
Detectable region
> 95% confidence
V(ϕ) = Λ_0^4 exp(Λ^n/ϕ^n); n =1
Λ_0 = 2.4 × 10^-3 eV
10^-4
10^-3
10^-2
10^-1
Energy scale of potential: Λ (eV)



Predictions for New Experiment Proposed by Lamoreaux
M = M_Pl
ΔF_φ >0.05pN
ΔF_φ > 0.1pN
ΔF_φ > 0.01 pN
(b)
Chameleon to matter coupling: M (GeV)
10^20
10^15
10^10
10^5
0
1
2
3
4
5
6
7
8
Slope of Potential (n)

Chameleons Coupled to Photons

- Chameleons may couple to electromagnetism:

$$\mathcal{L}_{\text{optics}} = \frac{e^{\phi/M}}{g^2} F_{\mu\nu} F^{\mu\nu}$$

- Cavity experiments in the presence of a constant magnetic field may reveal the existence of chameleons. The chameleon mixes with the polarisation orthogonal to the magnetic field and oscillations occur

- The coherence length $$z_{\text{coh}} = \frac{2\omega}{m^2}$$

depends on the mass in the optical cavity and therefore becomes pressure and magnetic field dependent:

$$\rho = \rho_m + \frac{B^2}{2}$$

- The mixing angle between chameleons and photons is: $$\theta = \frac{B\omega}{Mm^2}$$

Astrophysical Photon-ALP Mixing

- Magnetic fields known to exist in galaxies/galaxy clusters
- Made up of a large number of magnetic domains
  - field in each domain of equal strength but randomly oriented
- ALP mixing changes astrophysical observations
  - Non-conservation of photon number alters luminosity
  - Creation of polarisation in initially unpolarised light

Strong Mixing in Galaxy Clusters

- Strong Mixing:

$$P_{\gamma\leftrightarrow\phi} \approx \sin^2\left(\frac{L_{dom}B}{2M}\right)$$

- Galaxy Cluster
  - Magnetic field strength $B \approx 1 - 10\mu\mathrm{G}$
  - Magnetic coherence length $L \sim \mathrm{kpc}$
  - Electron density $n_e \sim 10^{-3}\,\mathrm{cm}^{-3}$
  - Plasma frequency $\omega_{\mathrm{pl}} \sim 10^{-12}\,\mathrm{eV}$
  - Typical no. domains traversed $N \approx 100 - 1000$
- Strong mixing if

$$N P_{\gamma\leftrightarrow\phi} \gg 1$$

$$\Delta = m_{\mathrm{eff}}^2 L/4\omega$$

$$N\Delta \lesssim \pi/2$$

Effects of Strong Mixing on Luminosity

- After passing through many domains power is, on average, split equally between ALP and two polarisations of the photon. **Average** luminosity suppression = 2/3
- Difficult to use this to constrain mixing because knowledge of initial luminosities is poor
- Single source of initial polarisation p_0, photon flux after strong mixing is

$$I_{\mathrm{f}}^{(\gamma)}(K_1, K_2, p_0) = \left[1 - (1 - p_0)K_1^2 - p_0 K_2^2\right] I^{(\mathrm{tot})}$$

$$K_i \sim U(-1, 1)$$

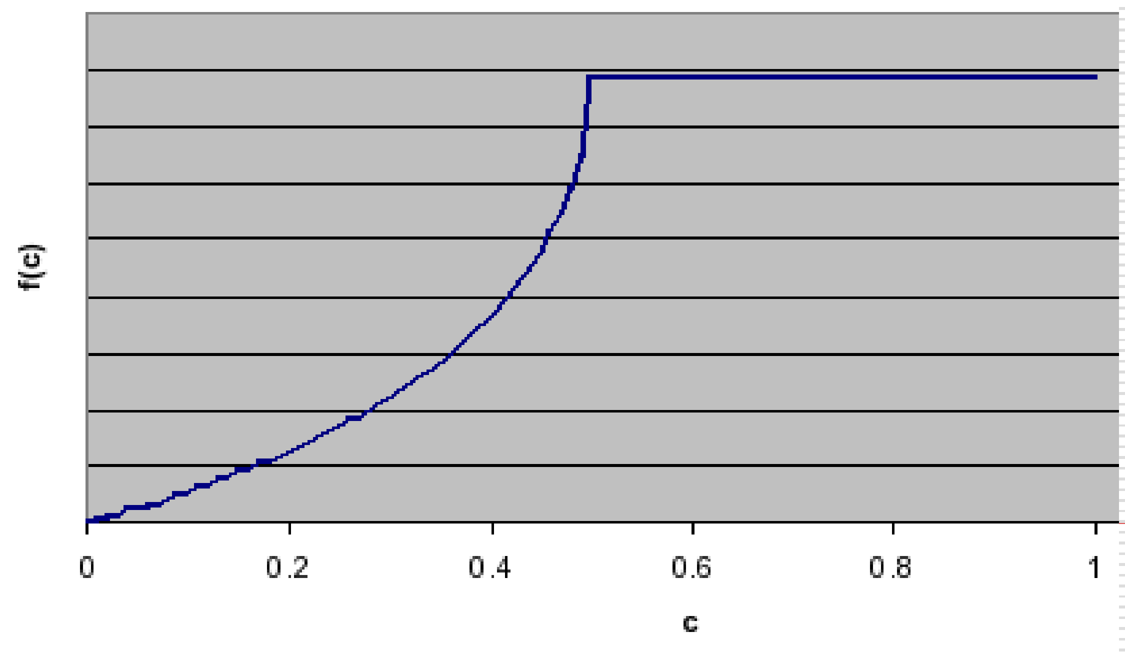
  - If $C \equiv I_{\mathrm{f}}^{(\gamma)}/I^{(\mathrm{tot})}$, averaged over many paths $\bar{C} = 2/3$

Effects of Strong Mixing on Luminosity

- Probability distribution function : C

$$f_C(c;p_0) = \frac{1}{\sqrt{1-p_0^2}}\left[\tan^{-1}\left(\sqrt{a}\left(1-\frac{2c_+}{1+p_0}\right)^{-1/2}\right) - \tan^{-1}\left(\sqrt{a}\left(1-\frac{2c_-}{1-p_0}\right)^{1/2}\right)\right]$$

$a = (1-p_0)/(1+p_0)$ $\quad$ $c_\pm = \min(c, (1 \pm p_0)/2$



Luminosity Relations

- Empirically established relations between high frequency luminosity and some feature at lower frequency
  - e.g. peak energy, or luminosity
- Standard relation $$\log_{10} Y_i = a + b \log_{10} X_i + S_i.$$

High frequency feature

Low frequency feature

  - If Gaussian noise $S_i = \sigma \delta_i \qquad \delta_i \sim N(0,1)$
  - If strong ALP-photon mixing in addition $$S_i = \sigma \delta_i - \log_{10} C_i + \mu,$$
  - Detection possible if Gaussian component smaller

Active Galactic Nuclei

- Strong correlation between 2 keV X-ray luminosity and optical luminosity (~5eV)
- Use observations of 203 AGN from COMBO-17 and ROSAT and SDSS surveys (z=0.061-2.54)
- Likelihood ratio $0 < p_0 < 0.4$
  - r≈14 Assuming initial polarisation
  - r>11 Allowing all polarisations
- Is this really a preference for ALPsm? Or just an indication of more structure in the scatter?

Fingerprints

- 10^5 bootstrap resamplings (with replacement) of the data - all samples 203 data points
- Compute the central moments of the data

$$k_m(\{s_i\}) = \left(\frac{1}{N_p}\sum_i s_i^{*m}\right)^{\frac{1}{m}}$$

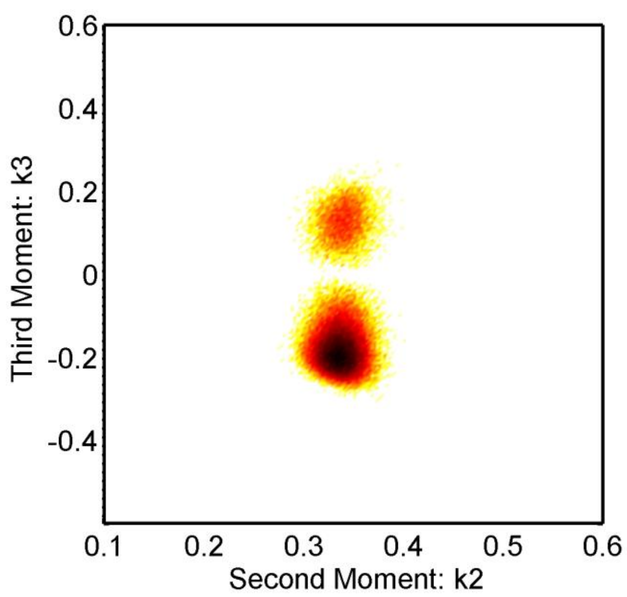
$$D = \langle x_i \equiv \log_{10} X_i, y_i \equiv \log_{10} Y_i \rangle$$

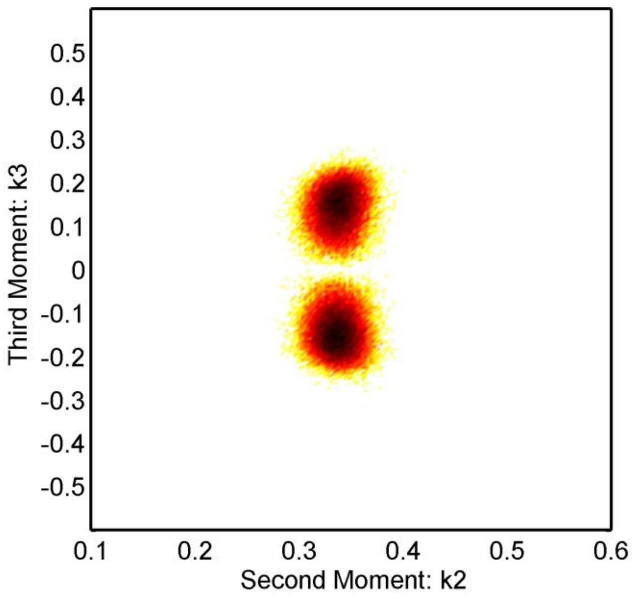
$$s_i^* = y_i^* - (a^* + b^* x_i^*)$$

  - k_2 is the standard deviation
  - k_3^3/k_2^3 is the skewness of the data
  - Compare this with simulations of the best fit Gaussian and ALPsm models

Fingerprints of the Data

- Typical k_2-k_3 plots for a simulated best-fit Gaussian scatter model with σ_0 = 0.34 are:

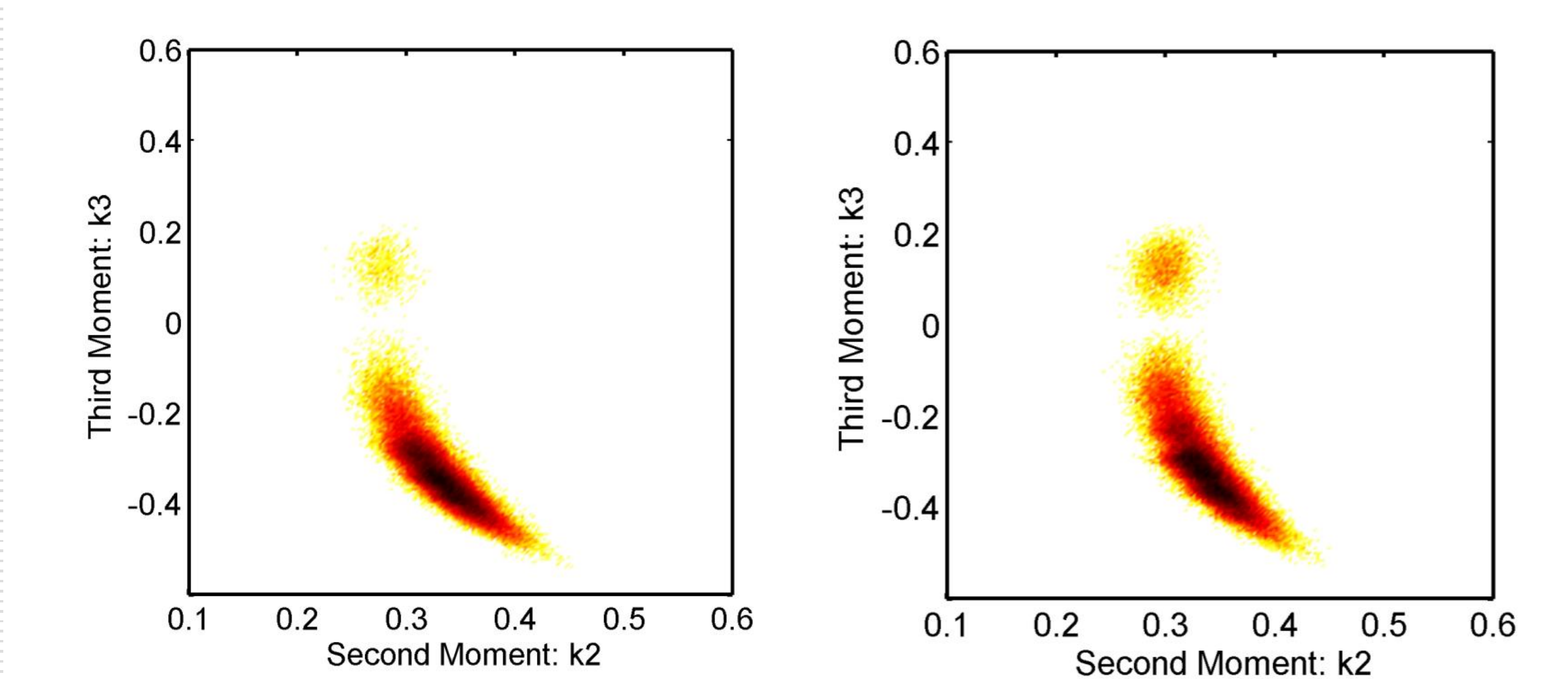




- Typically there are two roughly symmetric peaks, on the line $k_2 \sim 0.34$

Fingerprints of the Data

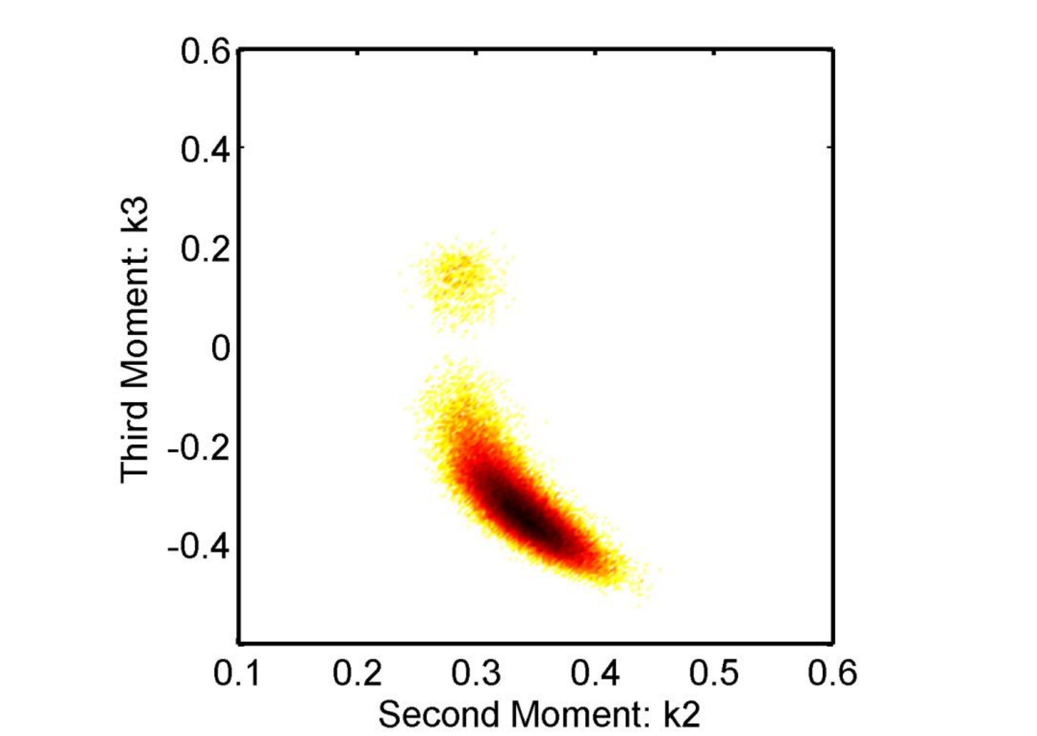
- In the ALPsm model there is more variation. Some times the simulated data is dominated by the Gaussian noise and one finds similar plots as before, and other times one sees:



- The key feature is the long tail.

AGN Fingerprint

- Constructing the same plot for the AGN data we find:



  - There is a clear qualitative similarity between this as the plots from simulated ALP data sets.
  - Suggestive that ALP strong mixing may be responsible for the scatter.

CMB – Chameleonic SZ Effect

- Can we see an effect in the CMB?
- When passing through galaxy clusters

the chameleon-photon conversion gives a `chameleonic Sunyaev-Zel'dovich effect'

(Inverse Compton scattering of photons off electrons in galaxy cluster.

- Boosts CMB intensity at high frequencies and causes a decrement at low frequencies.
- Observed change in intensity is $O(10^{-3})$

CMB intensity modification due to chameleon-photon mixing

When CMB photons travelled through galaxy clusters there is a mixing with chameleons due to the magnetic field in the cluster. This changes the photon intensity $I_f \approx I_0 \left(1 - \mathcal{P}_{\gamma\leftrightarrow\phi}(L)\right)$ resulting in a temperature shift

$$\frac{\Delta T_{\rm CZE}}{T_0} = \frac{1 - e^{-x}}{x}\frac{\Delta I_{\rm CZE}}{I_0} = \frac{e^{-x} - 1}{x}\bar{\mathcal{P}}_{\gamma\leftrightarrow\phi}(L).$$

where $x = \omega / kT_0$. We can evaluate this.

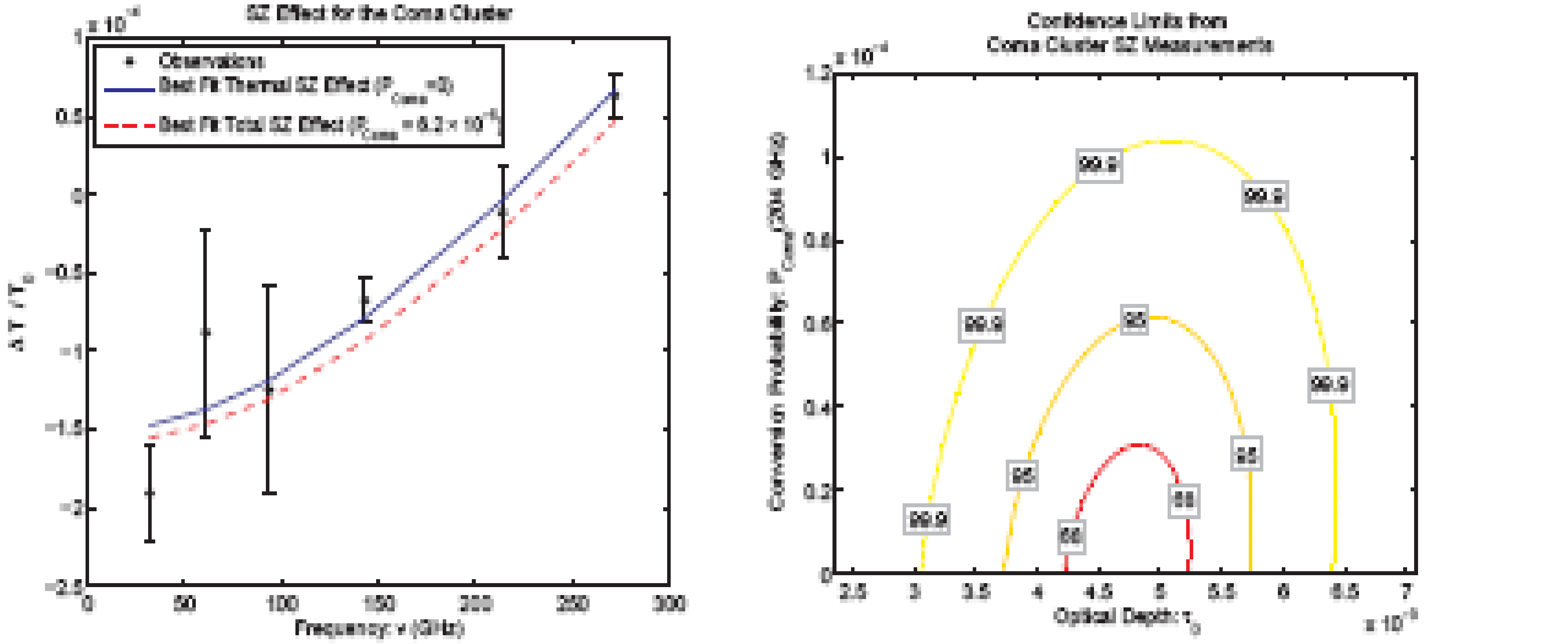


Fig 2: *Left: a plot of the existing SZ measurements for Coma, with best fit lines (a) assuming only a thermal SZ effect (solid blue) and (b) including the 95% confidence upper limit on the chameleon effect (dashed red). Right: confidence limits on the optical depth and conversion probability at* 204 GHz.

Radial profile of the SZ effects

The magnetic field strength, electron density and coherence length are expected to decrease with distance from the cluster centre. We assume the simplest scaling

$$n_e(r) = n_0 \left(1 + \frac{r^2}{r_c^2}\right)^{-3/2}; \quad L_{\rm coh} \propto n_e^{-1/3}; \quad B \propto n_e^{\eta}$$

where $\eta = 0.9$ is suggested from observations. The chameleonic SZ effect is predicted to dominate over the thermal SZ towards the edges of the cluster. Future radial SZ measurements at higher frequencies (≥ 90GHz) could further constrain the chameleon model.

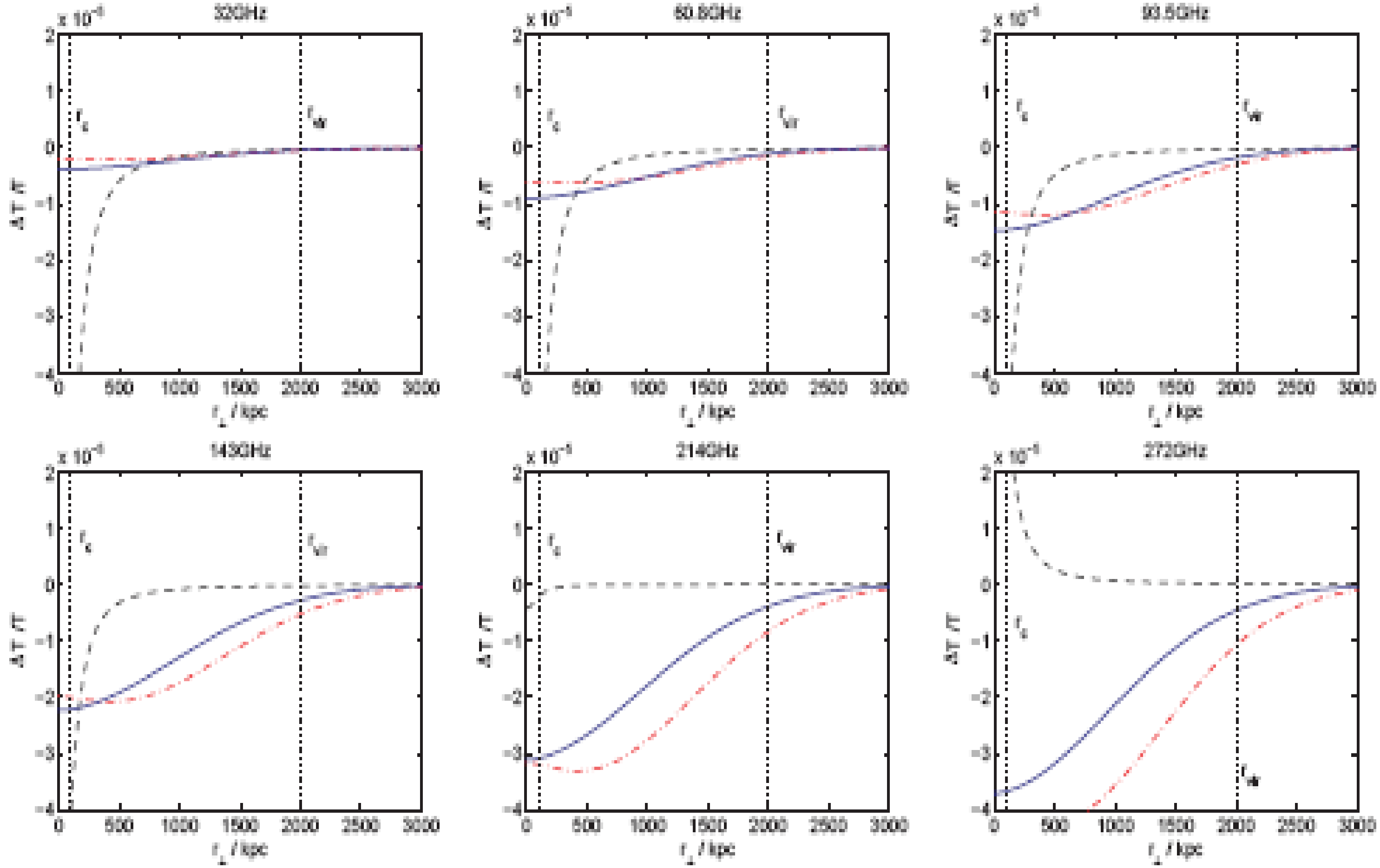


FIG. 4: Chameleonic (red dot-dashed line for $\alpha = 2$ and solid blue line for $\alpha = -5/3$) and thermal (dashed black line) SZ intensity decrement profiles for a simulated cluster at six different frequencies (take to be the same as the frequency bands in the Coma cluster measurements). The simulated cluster has $\mathcal{P}_{\gamma\leftrightarrow\phi}(204\,\mathrm{GHz}) = 3\times10^{-5}$, $\tau_0 = 5\times10^{-3}$ through the centre of the cluster, $r_c = 100\,\mathrm{kpc}$ and $r_{\rm vir} = 2\,\mathrm{Mpc}$. We assume $\eta = 0.9$ and $\beta = \gamma = 1$.

Large Scale Structures

Linear Growth factor

At the background level, chameleon models and their siblings the f(R) models behave like a pure cosmological constant.

Fortunately, this is not the case at the perturbation level where the growth factor evolves like:

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}H^2\left(1 + \frac{2\beta^2}{1 + \frac{m^2 a^2}{k^2}}\right)\delta = 0$$

The new factor in the brackets is due to a modification of gravity depending on the comoving scale k.

This is equivalent to a scale dependent Newton constant.

Everything depends on the comoving Compton length:

$$\lambda_c = \frac{1}{ma}$$

Gravity acts in an usual way for scales larger than the Compton length

$$\delta \sim a$$

Gravity is modified inside the Compton length with a growth:

$$\delta \sim a^{\frac{\nu}{2}}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta^2)}}{2}$$

Everything depends on the time dependence of m(a). If m is a constant then the Compton length diminishes with time. So a scale inside the Compton length will eventually leave the Compton length

Modified gravity $\Rightarrow$ General Relativity

z=z*

On the other hand, for chameleons the Compton length increases implying that scales enter the Compton length.

General Relativity $\Rightarrow$ Modified gravity

z=z*

Growth index

On either sides of the Compton length, we are interested in the growth function of CDM (and also baryons):

$$f(\ln a, k) = \frac{d \ln \delta_B}{d \ln a}$$

Modified gravity implies that the growth is altered:

$$f(\ln a, k) = (1 + g_B(\ln a)) f_0(\ln a), \qquad f_0 \sim \Omega_m^{0.55}$$

The deformation is a slowly varying function:

$$g_B \approx -\frac{5}{4} + \sqrt{\frac{25}{16} + \frac{3}{2}\alpha_{BB}(x)}$$

$$x = \frac{am}{k}, \quad \alpha_{BB}(x) = \frac{2\beta^2}{1+x^2}$$

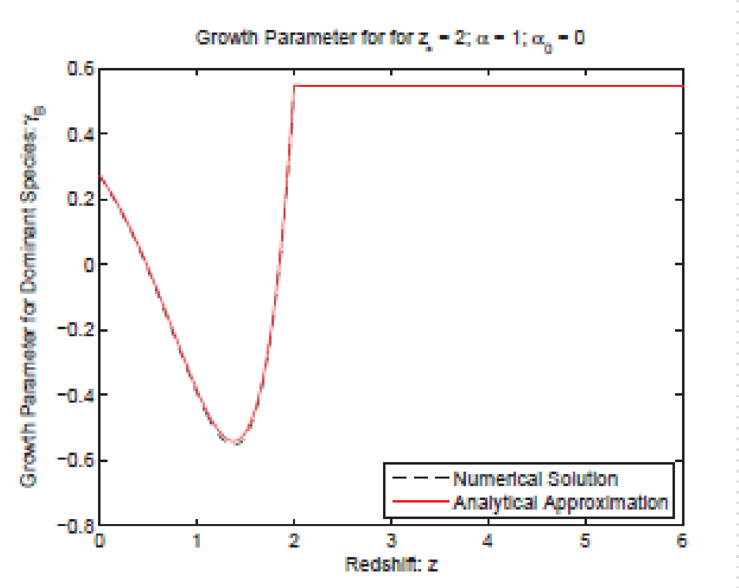
The effective growth index is corrected:

$$\gamma_B \approx 0.55 + \frac{\ln(1+g_B)}{\ln \Omega_m}$$

CDM growth index

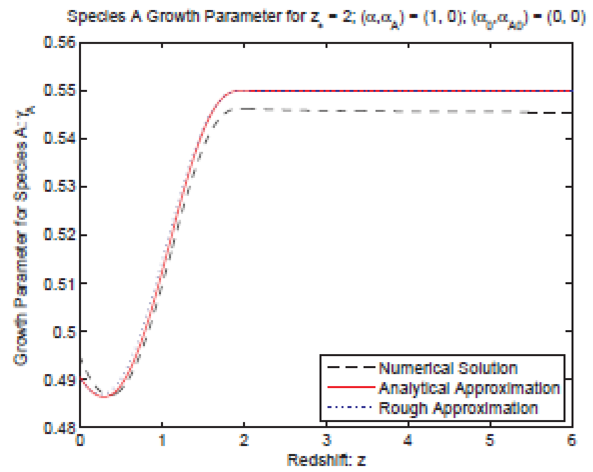
The growth index of CDM has a pronounced deviation from General Relativity.

Its behaviour depends on both z* and the strength of the coupling α_{BB}



Baryonic Growth index

The difference between the baryonic growth and the CDM growth is enhanced when baryons are not coupled to the scalar field.



Slip Function

The metric in the Jordan frame:

$$ds^2 = e^{2\beta\kappa_4\phi}a^2(\eta)(-(1+2\phi_N)d\eta^2+(1-2\phi_N)dx^2)$$

$$\psi = \phi_N + \kappa_4\beta\phi_N, \quad \Phi_N = \phi_N - \kappa_4\beta\phi$$

One may also define a slip function by correlating weak lensing and peculiar velocities:

$$\eta_\theta \equiv \frac{\Phi_N}{\psi_N} = \frac{1-2\alpha^2}{1+2\alpha^2}$$

Many other possibilities with slip functions, by studying weak lensing, ISW, growth of structures etc….

An Example: the radion

The distance between branes in the Randall-Sundrum model:

$$A(\phi) = \cosh \frac{\phi}{\sqrt{6}}$$

where

$$R = \frac{1}{k} \ln \tanh \frac{\phi}{\sqrt{6}}$$

Gravitational coupling:

$$\alpha_\phi = \frac{1}{\sqrt{6}} \tanh(\frac{\phi}{\sqrt{6}})$$

close branes:

$$A(\phi) = \exp \frac{\phi}{\sqrt{6}}$$

constant coupling constant

$$\alpha_\phi = \frac{1}{\sqrt{6}}$$

The Dilaton

String theory in the strong coupling regime suggests that the dilaton has a potential:

$$V(\phi) = V_0 e^{-\phi} + \ldots$$

Damour and Polyakov suggested that the coupling should have a minimum:

$$A(\phi) \approx 1 + \frac{A_2}{2}(\phi - \phi_0)^2 + \ldots$$

The coupling to matter becomes:

$$\alpha_\phi \approx A_2(\phi - \phi_0)$$

In the presence of matter, the minimum plays the role of an attractor:

$$\phi - \phi_0 \approx \frac{1}{1 + A_2 \frac{\rho}{V_0}}$$

The coupling becomes:

$$\alpha_\phi \approx \frac{A_2}{1 + A_2 \frac{\rho}{V_0}}$$

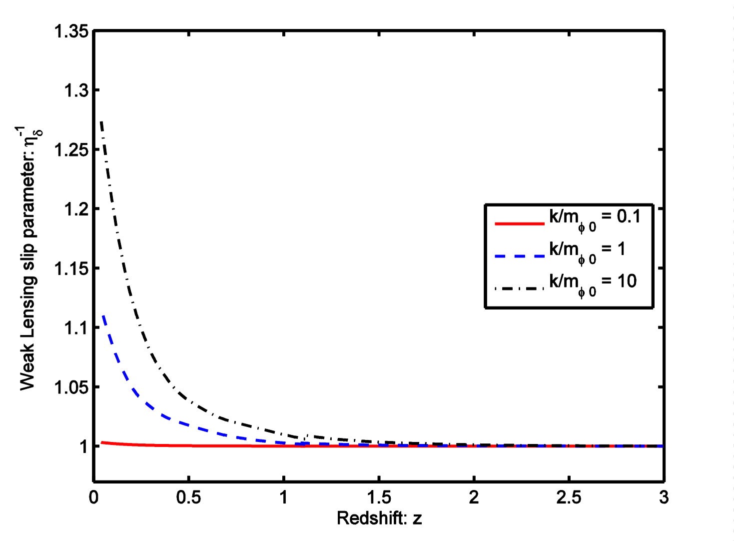
Three regimes:

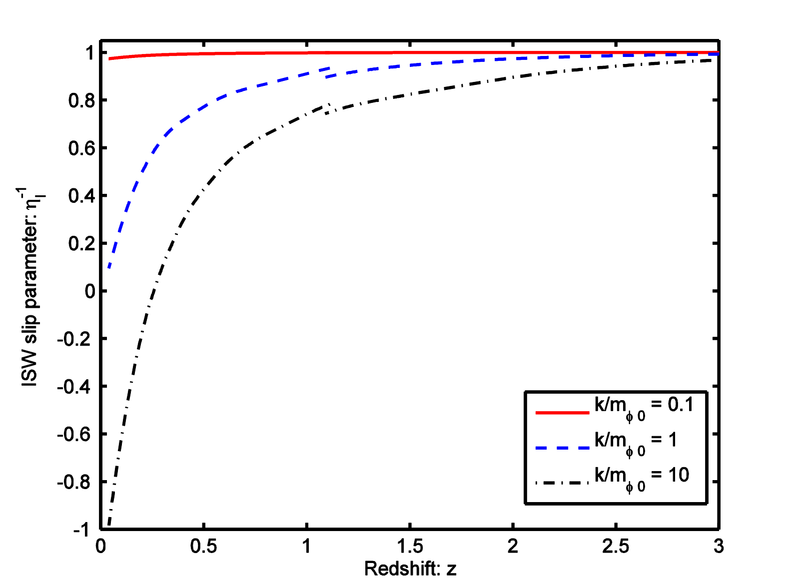
i) early in the universe, large density: small coupling.

ii) recent cosmological past: large scale modification of gravity.

iii) collapsed objects: small coupling.

The Dilatonic case





Conclusions

- Chameleon theories are intriguing and lead to new physics
- The can explain the observed acceleration of the universe and lead to experimental predictions
- Hints of such theories have been seen in AGNs and starlight polarisation
- Experimentally viable f(R) theories are indistinguishable from a cosmological constant
- Effects on large scale structure leads to exciting prospects for the future